TESSO: An Analytical Tool for Characterizing Aggregate Interference and Enabling Spatial Spectrum Sharing

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Abstract—Radio propagation models play a crucial role in realizing effective spectrum sharing. Unlike propagation models that do not use the exact details of terrain, terrain-based propagation models are effective in identifying spatial spectrum sharing opportunities for the secondary users (SUs) around an incumbent user (IU). Unfortunately, terrain-based propagation models, such as the Irregular Terrain Model (ITM) in point-to-point (PTP) mode, are computationally expensive, and they require precise geo-locations of the SUs. Such requirements render them challenging, if not impractical, to implement in real-time applications, such as geolocation database (GDB)-driven spectrum sharing. To address this problem, we propose a pragmatic approach called Tool for Enabling Spatial Spectrum Sharing Opportunities (TESSO). TESSO characterizes the aggregate interference caused by the SUs and identifies spatial spectrum sharing opportunities effectively. It is computationally efficient, and does not require precise geo-locations of the SUs. Our results show that TESSO provides the same level of interference protection guarantee to the IU as that offered by the terrain-based models. TESSO can be implemented in GDB-driven spectrum sharing ecosystems for effectively exploiting spatial spectrum sharing opportunities.


I. INTRODUCTION

Wireless spectrum is a valuable resource, and securing its optimal use, through spectrum sharing, is a key to spurring technological innovations as well as economic growth. The importance of radio frequency spectrum to the national economy has motivated regulatory agencies and the federal government to aggressively push forward with a number of spectrum reform initiatives [1]–[4]. In spectrum sharing, two types of stakeholders share the spectrum: (i) incumbent users (e.g., licensed users) (IUs), and (ii) secondary users (e.g., unlicensed users) (SUs). IUs have priority-access rights to their licensed spectrum whereas SUs are allowed to opportunistically access the shared spectrum provided that they do not cause harmful interference to the IUs. In general, spectrum sharing between IUs and SUs can be realized in three domains, namely, time, frequency, and space. In this paper, we focus our discussions on spatial spectrum sharing.

The accurate prediction of radio propagation path loss plays a crucial role in realizing effective spectrum sharing, particularly in the context of geolocation database (GDB)-driven spectrum sharing. In GDB-driven spectrum sharing, such as sharing dictated by a Spectrum Access System (SAS)\(^1\), a spectrum management entity first computes the expected co-channel interference that a prospective SU may cause to the IU. To compute the expected interference, the SAS uses an appropriate radio propagation path loss model\(^2\) as well as information such as the SU’s and the IU’s geo-locations, their antenna parameters and the SU’s transmit power. The result of this analysis is then combined with information about the IU’s interference protection criteria to compute spectrum availability at the prospective SU’s location. If the estimated aggregate interference to the co-channel IU is below its interference tolerance threshold, the SU is allowed to transmit in the co-channel; otherwise not. To fully reap the benefits of spectrum sharing, an accurate propagation analysis is desired. A propagation analysis that over-estimates path loss between the SU and the IU will under-estimate the potential for co-channel interference, providing inadequate interference protection for the IU. In contrast, an analysis that under-estimates the path loss will unnecessarily preclude SUs from taking advantage of fallow spectrum.

Often times, in spectrum sharing, multiple SUs share the spectrum with an IU. For example, in the three-tiered sharing architecture of the U.S. 3.5 GHz band, multiple Priority Access Licensed (PAL) users and General Authorized Access (GAA) users share the band with an incumbent ship-borne radar [2]. Here, the interference power received at the IU is not just the interference caused by a single SU, but in fact, it is the aggregate interference caused by multiple SUs. To ensure

\(^1\)The Spectrum Access System is the term used in the recent FCC and NTIA documents to denote a network of databases and spectrum managers deployed to enable dynamic spectrum sharing.

\(^2\)A radio propagation model is an empirical mathematical formulation for the characterization of radio propagation path loss as a function of frequency, distance and other parameters.

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harmonious coexistence, the spectrum manager of a purely GDB-driven spectrum sharing ecosystem should estimate the aggregate interference, and allow an entrant SU to transmit in the co-channel only if doing so does not cause harmful interference to the IU.

Studies have shown that the use of terrain-based propagation models, such as Irregular Terrain Model (ITM) in point to point (PTP) mode\(^3\), improves the efficacy of spectrum sharing because such models accurately estimate the path loss in a communication link [5]. For example, in June 2015, the National Telecommunications and Information Administration (NTIA) published a report that shows that the exclusion zone\(^4\) of IUs in the 3.5 GHz band can be reduced by up to 70% when legacy propagation models are replaced by terrain-based propagation models such as ITM-PTP model [6].

Unfortunately, using ITM-PTP for characterizing aggregate interference caused by multiple SUs might not be viable for several reasons. First, ITM-PTP model is computationally intensive and data hungry due to the consideration of detailed environmental parameters in path loss computations. Therefore, when \(N\) SUs are likely to share the spectrum with an IU—i.e., when \(N\) interferers possibly contribute to the aggregate interference at the IU—computing the aggregate interference requires \(N\) ITM-PTP path loss computations, which requires very long processing time when \(N\) is large. Second, ITM-PTP requires accurate geo-locations of SUs which might not be available in all spectrum sharing scenarios. For example, in many cases, the aggregate interference from eNodeBs as well as user equipment (UE) may need to be considered, but acquiring the exact geo-locations of UEs may not be feasible when they are mobile. Also, having a centralized sharing entity know exact locations of all SUs creates privacy problems that may otherwise be avoided.

Due to the aforementioned limitations of ITM-PTP, an alternative approach, or an analytical tool, for characterizing the aggregate interference is desirable in some spectrum sharing scenarios. The tool should be able to accurately estimate the aggregate interference in a computationally efficient manner, and it should be effective even when precise geo-locations of SUs are not available. In this paper, we propose an analytical tool that we refer to as Tool for Enabling Spatial Spectrum Sharing Opportunities (TESSO). TESSO is a tool that can be employed by a central spectrum management entity (such as a SAS) to perform real-time estimates of the SUs’ aggregate interference power, which is the key parameter needed to perform spectrum access control (i.e., control which and how many SUs are allowed to access spectrum). Note that the objective of our work is not to specifically compare TESSO against ITM-PTP. Instead, what we try to demonstrate is that a computationally efficient tool, such as TESSO, can be used to characterize the aggregate interference in spectrum sharing, and such model, if used appropriately, provides almost as effective results as existing techniques, such as the ITM-PTP.

The main features of TESSO are summarized below:

- TESSO enables us to analytically model the aggregate interference caused by SUs (as measured at an IU). Using the mathematical model for aggregate interference, TESSO effectively facilitates GDBs to identify spatial spectrum sharing opportunities around an IU.
- TESSO is computationally efficient, and it can be implemented by a SAS ecosystem in real-time to compute the maximum number of SUs, say \(N\), that can be safely allowed to co-exist with an IU. To compute \(N\), TESSO does not require the precise geo-locations of the SUs.
- The performance of TESSO, in terms of spectrum utilization and incumbent protection, is comparable to that of computationally intensive terrain-based propagation models, such as ITM-PTP.

The road map of the rest of the paper is as follows. In Section II, we discuss the preliminaries. In Section III, we demonstrate the effectiveness of terrain-based propagation model in enabling spatial spectrum sharing opportunities and illustrate how ITM-PTP mode ensures the protection of IU from aggregate interference caused due to SUs. Later, in Section IV, we introduce and provide details of TESSO. Case studies for evaluating the performance of TESSO are presented in Section V. Finally, Section VI concludes the paper.

II. PRELIMINARIES

A. Irregular Terrain Model

The ITM is a radio propagation model, which predicts tropospheric radio transmission loss over irregular terrain for a radio link [7]. It is designed for use at frequencies between 20 MHz and 20 GHz, and for path lengths between 1 km and 2000 km. ITM estimates radio propagation loss as a function of distance and other factors such as variables in time and space. Radio propagation loss is computed based on electromagnetic theory, and signal loss variability expressions are derived from comprehensive sets of measurements. ITM is both data and computationally intensive. There are two modes of operation of ITM: (i) Area prediction (ITM-AP) mode, and (ii) Point-to-point (ITM-PTP) mode.

In the ITM-AP mode, the term “area” is described by the terrain irregularity parameter, \(\Delta h\), and the effective antenna heights of the system [8]. Based on \(\Delta h\) and other parameters, the ITM-AP mode predicts the path loss between any two given points. In contrast, the ITM-PTP mode takes into account the actual obstructions between the transmitter and the receiver. To make its predictions, the ITM-PTP mode incorporates the principal determinants of radio propagation over irregular terrain paths, which include the amount by which the direct ray clears terrain prominences, position of terrain obstacles and their degree of roundness, apparent Earth flattening due to atmospheric refraction, etc. [9].

The ITM-PTP mode relates the statistical variance of terrain elevations to classical diffraction theory, and predictions

\(^3\)The ITM in PTP mode is the most popular terrain-based propagation model in use today.

\(^4\)An exclusion zone is the area around an IU where co-channel/adjacent-channel transmissions from SUs are prohibited.
made by the model agree closely with the measured data. Comparison with actual measurements validates that path loss values calculated by the ITM-PTP mode are quite accurate; and moreover that the accuracy of the ITM-PTP mode is as good as or better than that achieved by alternative procedures [9]. In this paper, we synonymously use the term terrain-based propagation model to refer to the ITM-PTP model.

B. Aggregate Interference

When multiple SUs share the spectrum with an IU, the interference power received at the IU is not just the interference caused by a single SU, but it is the aggregate interference caused by multiple SUs. A successful design and deployment of dynamic spectrum access, therefore, requires an accurate model for characterizing the aggregate interference. This characterization feeds into the design of transmission policies for SUs and protects IUs from SU-generated interference.

To characterize the performance of IUs in dynamic spectrum access, a detailed analysis of the aggregate interference needs to be done. In practical networks, a multitude of factors must be considered together in order to arrive at an accurate statistical model for the aggregate interference. Aggregate interference depends on propagation characteristics of the channels between the SUs and the IU, such as path loss, shadowing and fading, and also on the transmit power control scheme used by the SUs. Terrain characteristics in the link between the SUs and the IU also affect the distribution of aggregate interference. Furthermore, the number of SUs that transmit and their locations, themselves are random variables and affect the aggregate interference.

Given the importance of aggregate-interference modeling in dynamic spectrum access, researchers have studied this topic extensively in the past few years. Some works focus on developing statistical interference models, while others provide exact analysis and performance bounds. For example, Bhattacharjya et al. in [10] derived an expression for the aggregate interference by considering a path loss model that is based on exponential path loss and log-normal shadowing. They showed that the aggregate interference from a fixed number of SUs, distributed uniformly over a region, can be modeled as a log-normal random variable. In [11], the authors used the method of log-cumulants to approximate the distribution parameters of the aggregate interference. Ghasemi and Sousa, in [12], developed a statistical model of interference aggregation in spectrum-sensing cognitive wireless networks by explicitly taking into account the random variations in the number, location and transmitted power of SUs as well as the propagation characteristics. The authors of [13] suggest that, for arbitrarily-shaped network regions, the shifted log-normal distribution provides the overall best approximation for the aggregate interference, especially in the distribution tail region. In general, these models for aggregate interference are useful not only in characterizing the performance of dynamic spectrum access networks, but also in designing protection zones around an IU [10], [14], deploying cognitive radios [15], managing spectrum access control, etc.

C. Exclusion Zones

The notion of an exclusion zone (EZ) is a static spatial separation region defined around an IU, where co-channel and/or adjacent-channel transmissions by SUs are prohibited. EZs are the primary ex-ante mechanism employed by regulators to protect IUs from harmful interference caused by transmissions from SUs. The legacy EZs are conservative and static. The notion of a static exclusion zone implies that it has to protect IUs from the union of all likely interference scenarios, resulting in a worst-case and very conservative solution [10]. Since SU operations are prohibited inside an EZ, the conservative design of EZs unnecessarily limits the SUs’ spatial spectrum-access opportunities. Recently, in its Notice of Proposed Rule Making (NPRM) [16], the Federal Communications Commission (FCC) acknowledged that the size of an EZ could be significantly reduced if a realistic propagation model could be used in conjunction with a mechanism to monitor the aggregate interference caused by SU transmissions.

III. ILLUSTRATIVE EXAMPLE OF ITM-PTP MODE

Before introducing TESSO in the next section, here, we provide an illustrative example to demonstrate the effectiveness of a terrain-based propagation model in discovering spatial spectrum sharing opportunities. Specifically, we compare ITM-PTP mode—a terrain-based propagation model— against the ITM-AP mode—a model that does not use details of terrain in the path loss computations. Later, in Section V, we use these results as a benchmark to evaluate the relative effectiveness of TESSO in identifying spatial spectrum sharing opportunities.

Let us define an analysis area of approximately 40,000 square kilometers centered at (36°, −77°) latitude-longitude as shown in Figure 1(a). An IU, operating in the 3550 MHz band, is located at the center of the analysis area. Let us divide the analysis area into square grids, each with a side length of 0.01° (roughly 1 km). Now, using ITM-AP mode along with the parameters listed in Table I, radio propagation loss is computed from the center of each grid to the IU. For computing the ITM-AP path loss, we use the terrain irregularity parameter, \( \Delta h = 90 \) m. The resulting path loss map (in dB) is shown in Figure 1(a). The oval shape of the path loss contours is attributed to the fact that ITM-AP mode does not consider the exact details of terrain to compute the path loss in the point to point link. Rather, it uses the average terrain characteristics—defined by \( \Delta h \)—for path loss computations. For a given \( \Delta h \), path loss from a grid to the IU is a function of distance, but not of the actual terrain in the link connecting the two points.

Figure 1(b) shows the path loss contours when ITM-PTP mode is used to compute the propagation loss for the same

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### TABLE I: ITM parameters used in our analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Radio frequency, ( f )</td>
<td>3550 MHz</td>
</tr>
<tr>
<td>Surface refractivity</td>
<td>301 N-units</td>
</tr>
<tr>
<td>Dielectric constant of ground</td>
<td>15</td>
</tr>
<tr>
<td>Conductivity of ground</td>
<td>0.0005 S/m</td>
</tr>
<tr>
<td>Radio climate</td>
<td>Continental temperature</td>
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map. We used the same parameters as outlined in Table I. For extracting the terrain details in the path loss computations, we used the Global Land One-km Base Elevation (GLOBE) database [17]. GLOBE is an internationally designed, developed, and independently peer-reviewed global digital elevation model, at a latitude-longitude grid spacing of 30 arc-seconds (3′). Unlike ITM-AP mode, the path loss contours obtained by using ITM-PTP mode are highly irregular in shape. The irregularities occur from the fact that specific terrain details in the point to point link are considered while computing the propagation loss. Furthermore, the path loss predicted by the ITM-PTP mode is often larger and more accurate than that predicted by the ITM-AP mode. By comparing Figures 1(a) and 1(b), it is evident that ITM-PTP mode’s ability to produce accurate path loss estimates can be utilized to identify spatial sharing opportunities, albeit such an approach would incur a high computational cost.

Suppose that a SU transmits with power $P_{ts} = 30$ dBm, the receiver antenna gain of the SU is $G_r = 0$ dB, the transmitter antenna gain of the SU is $G_t = 0$ dB, and the interference tolerance threshold of the IU is $I_{th} = -120$ dBm, then the minimum required path loss, $P_{rl_{min}}$, for protecting the IU from interference caused by a single SU, $I_{SU}$, is computed as,

$$P_{rl_{min}} = P_{ts} + G_t + G_r - I_{th} = 140$$ dB.

In general, multiple SUs may operate around an IU. In order to ensure that the aggregate interference caused by multiple concurrently-transmitting SUs does not exceed $I_{th}$, a conservative margin of $\Delta P_L = 10 - 20$ dB is often added to $P_{rl_{min}}$ [18]. For example, using $\Delta P_L = 20$ dB with the aforementioned parameters results in $P_{rl_{min}} = 160$ dB. Then, using $P_{rl_{min}}$, an EZ can be defined around an IU where SU transmissions are prohibited. Figure 1(c) shows the resulting EZ. The black oval represents the EZ when the ITM-AP mode is used to estimate the path loss, whereas the irregular blue contour represents the EZ when ITM-PTP mode is used. We refer to the spatial region that is inside the EZ defined by the ITM-AP mode, but outside the EZ defined by the ITM-PTP mode as the spatial white space (SWS). Based on the assumptions that were made above, an ITM-PTP mode can safely allow a limited number of SUs, say $N$, to operate inside a SWS without violating the IU’s interference protection requirement (i.e., the aggregate interference power received by the IU does not exceed the interference tolerance threshold).

Now, let us assume that IUs can operate without noticeable performance degradation if they are ensured a probabilistic guarantee of interference protection—i.e., an IU’s interference protection is prescribed as follows: the aggregate interference, $I_{agg}$, from SUs is below $I_{th}$ for $(1-\epsilon)$ fraction of the time, where $\epsilon$ is the probability that $I_{agg} > I_{th}$. That is,

$$P(I_{agg} \leq I_{th}) \geq 1 - \epsilon.$$  \hspace{1cm} (1)

Since radio signal propagation is inherently stochastic, the notion of a probabilistic guarantee for interference protection is widely accepted. For example, the coverage regions of TV stations are based on F-curves, which provide a probabilistic guarantee that the signal reception is above a threshold [19].

Before introducing TESSO in the next section, let us discuss a methodology, described by Algorithm 1, that can be employed by a SAS for evaluating spatial sharing opportunities in the SWS region. The proposed methodology is an approach for computing the maximum number of SUs that can be safely allowed to operate in the SWS region while satisfying Inequality (1). Algorithm 1 is based on the computationally intensive ITM-PTP model that requires the precise locations of SUs. Despite its high computational cost, the use of ITM-PTP model for path loss computations makes Algorithm 1 effective in accurately identifying the spatial spectrum sharing opportunities around an IU. In our analysis, the solution produced by Algorithm 1 represents the ground truth—i.e., it represents the true number of SUs that can be safely allowed in the SWS region. Later, in Section V, we use the solution produced by Algorithm 1 as a benchmark for comparing the relative effectiveness of TESSO.

The methodology described in Algorithm 1 is as follows. When an entrant SU requests for spectrum access, the SAS uses ITM-PTP path loss model to predict the interference that is likely to be caused by the SU at the IU and checks if the aggregate interference caused by SUs is below the IU’s interference tolerable threshold. For computing the interfer-
Algorithm 1 Evaluating spatial sharing opportunities in SWSs using ITM-PTP path loss model.

Input: Parameters listed in Table I, IU’s location, GLOBE data, \( I_{th} \), \( c \), \( P_{th} \), and SU queries from the SWS region.

Output: \( N \).

1: Initialize \( I_{agg} = 0 \), and \( N = 0 \).
2: for each SU query \( i \) do
3: if SU-SU coexistence criteria is satisfied then
4: Compute ITM-PTP path loss, \( P_{L_{ITM}} \), using IU’s location, SU’s location, GLOBE data and parameters listed in Table I.
5: Compute \( I_{SU} = P_{th} + G_t + G_r - P_{L_{ITM}} \), and \( I_{agg} = I_{agg} + I_{SU} \).
6: if \( I_{agg} < I_{th} \) then
7: Allow the \( i^{th} \) SU to transmit.
8: Update \( N = N + 1 \).
9: else if \( I_{agg} > I_{th} \) then
10: Generate a random number, \( u \), between 0 and 1.
11: if \( u \leq \epsilon \) then
12: Allow the \( i^{th} \) SU to transmit.
13: Update \( N = N + 1 \).
14: return \( N \).
15: else
16: Deny the \( i^{th} \) SU’s request to transmit.
17: return \( N \).
18: end if
19: end if
20: end if
21: end for

Fig. 2: Modeling a SWS region as annular sectors in case study 1. The color map represents ITM-PTP path loss.

IV. TESSO: A TOOL FOR ENABLING SPATIAL SPECTRUM SHARING OPPORTUNITIES

The use of realistic terrain-based propagation models is effective for identifying expanded SWS opportunities, while still providing IUs with appropriate interference protection. However, the computational complexity inherent in terrain-based propagation models and the requirement of precise geo-locations of SUs makes it challenging to implement them in real time systems, such as SAS-driven spectrum sharing. In this section, we describe an analytical tool—namely, TESSO—which is a mathematical framework for discovering SWS opportunities in a computationally efficient manner, while satisfying the IU’s protection requirement. TESSO is based on a simplified propagation model whose parameters are derived by characterizing the statistical properties of the radio propagation environment.

From Figure 1(c), we can notice that the SWS region is highly irregular in shape. In order to consider the irregularity of the SWS region while making TESSO analytically tractable, let us model the SWS region as a union of multiple annular sectors of an oval as shown in Figure 2. We define the term “SWS sector” to refer to each of these sectors. This sectorized SWS model strikes an appropriate compromise between modeling a realistic SWS region and limiting modeling complexity. Recall that the EZ boundary indicated by the black oval is conservatively large because it is computed using ITM-AP mode. Therefore, the interference emanated by SUs operating outside this conservative EZ boundary can be safely ignored. However, in the SWS sectors, it needs to be ensured that the statistics of aggregate interference caused by multiple SU transmissions does not violate the IU interference protection requirement. TESSO protects the IUs by enabling a SAS to carefully control the number of SUs that are allowed to transmit concurrently in the SWS sectors.

A. Interference from a Single SU

In order to analyze the interference caused by a single SU, \( I_{SU} \), at the IU, let us consider a SU operating inside a SWS sector. Also assume that SUs are uniformly distributed inside the SWS sector. Note that, in practical scenarios, SUs might be distributed non-uniformly, and such scenarios can be approximated by considering different SU densities in each SWS sector.

Let us consider a simplified propagation model with exponential path loss and log-normal shadowing. The path loss exponent, \( \gamma \), and the variance of log-normal shadowing, \( \sigma^2 \), for each sector can be estimated using a number of approaches; e.g., by using measurement data or by using estimates from more accurate propagation models. Using a simplified propagation model, the path loss, \( P_L \), to the IU from a SU located \( d \) meters away can be expressed as:

\[
P_L = a + b \log_{10} d + \psi,
\]

where, \( a = P_{L_{d0}} - b \log_{10} d_0 \), \( P_{L_{d0}} \) is the path loss at a reference distance, \( d_0 \), in dB, \( b = 10\gamma \), and \( \psi \) denotes the
shadowing coefficient which is lognormally distributed with mean $= 0$ and variance $= \sigma^2$.

Now, if $P_t$ denotes the transmit power of SU in dBm, then the interference power received by the IU receiver due to transmission from a SU is

$$I_{SU} = P_{ts} - P_d = P_{ts} - (a + b \log_{10} d + \psi). \quad (3)$$

When SUs are uniformly distributed in an annular sector, that is defined by $R_1$ and $R_2$ (the inner and the outer radius respectively), with the IU at the center, the distance between a SU and the IU is a random variable $D$ whose probability density function (PDF) is given by Equation (4) [20].

$$f_D(d) = \frac{2d}{R_2^2 - R_1^2}, \quad R_1 \leq d \leq R_2. \quad (4)$$

Strictly speaking, the outer and inner boundaries of a SWS sector are defined by arcs of concentric-ovals (because the Earth is not a perfect sphere), but, for simplicity, we approximate them as arcs of concentric-circles.

Finally, using transformation of random variables, the PDF of $I_{SU}$, denoted as $f_{I_{SU}}(i_{su})$, can be derived as [10]:

$$f_{I_{SU}}(i_{su}) = Ke^{\frac{2(\ln(i_{su} - a) - \ln 10)}{b(R_2^2 - R_1^2)} \left\{ \text{erf}(B) - \text{erf}(A) \right\}}, \quad (5)$$

where $K = \frac{b(R_2^2 - R_1^2) e^{\frac{2(\ln 10)^2}{\sigma^2} a^2}}{2b}$,

$$A = \frac{1}{\sqrt{2\sigma}} \left( P_{ts} - i_{su} - a - b \log_{10} R_2 + \frac{2\sigma^2 \ln 10}{b} \right)$$

and $B = \frac{1}{\sqrt{2\sigma}} \left( P_{ts} - i_{su} - a - b \log_{10} R_1 + \frac{2\sigma^2 \ln 10}{b} \right)$.

Equation (5) is valid for any SU operating in any SWS sector. When specific values of $a$, $b$, $P_{ts}$, $\sigma$, $R_1$ and $R_2$ pertaining to the $i$th SU operating in the $j$th SWS sector are plugged into Equation (5), the PDF of $I_{SU,i,j}$ is obtained. Here, $I_{SU,i,j}$ denotes the interference power at the IU induced by transmission from the $i$th SU operating in a randomly chosen location inside the $j$th SWS sector.

**Theorem 1.** For small $\omega$, where $\omega = \frac{R_2}{R_1}$, the PDF of $I_{SU}$ can be approximated as a log-normal distribution. The error in approximation increases monotonically with $\omega$.

**Proof.** Let us rewrite Equation (5) as follows,

$$f_{I_{SU}}(i_{su}) = K'\ g_1(i_{su})g_2(i_{su}), \quad (6)$$

where $g_2(i_{su}) = \text{erf}(g_3(i_{su})) - \text{erf}(g_3(i_{su}) - \frac{b \log_{10} \omega}{\sqrt{2}\sigma})$, and $g_3(i_{su})$ and $g_1(i_{su})$ are linear and exponential functions of $i_{su}$ respectively. Here, $K'$ is a non-negative constant.

From the definition of the erf function, the plot of $g_2(i_{su})$ can be approximated as a Gaussian PDF. This approximation is fairly accurate when $\frac{b \log_{10} \omega}{\sqrt{2}\sigma}$ is small. Restating this in terms of $\omega$, the Gaussian approximation holds true only for small values of $\omega$. Finally, because the product of an exponential kernel ($g_1(i_{su})$ has the kernel of an exponential distribution) and a Gaussian kernel ($g_2(i_{su})$ can be approximated to have the kernel of a Gaussian distribution) results in another Gaussian kernel, $f_{I_{SU}}(i_{su})$ is a Gaussian PDF.

In most spectrum sharing scenarios, $\omega$ is small because most of the SWSs are located near the $R_2$ boundary. Per Theorem 1, the distribution of $I_{SU}$ can be approximated as a log-normal distribution in such cases.

**B. Aggregate Interference**

To adequately protect IUs from the interference from multiple SUs, we must consider the distribution of $I_{agg}$, which is the summation of random variables, $I_{SU,i,j}$, and is defined as

$$I_{agg} = \sum_{j=1}^{N(j)} \sum_{i=1}^{S} I_{SU,i,j}, \quad (7)$$

where, $S$ denotes the total number of SWS sectors, and $N(j)$ is the total number of SUs in the $j$th SWS sector.

Since $I_{SU,i,j}$ can be approximated as a log-normal distribution, $I_{agg}$ is the summation of log-normal random variables. It has been shown that the summation of log-normal random variables can be approximated by another log-normal random variable. Among existing approximation methods [21]–[24], the Fenton-Wilkinson (FW) method [22] is a simple and computationally efficient algorithm for approximating the mean and variance of the resulting log-normal distribution. It provides a very good approximation in the tail region of the resulting complementary cumulative distribution function (CCDF) curve [25]. We employ the FW technique to approximate $I_{agg}$ because for very small values of $\epsilon$ (e.g., $0 \leq \epsilon \leq 0.1$), Inequality (1) represents the tail region of the CCDF of $I_{agg}$. Recall that we are interested in the tail region of the $I_{agg}$ distribution as dictated by Inequality (1).

The closed-form solutions provided by FW approximation are given in Equation (8) [22]:

$$\begin{align*}
\sigma_{agg}^2 &= \ln \left( \sum_{j=1}^{S} \sum_{i=1}^{N_j} \left( e^{\mu_{i,j} + \sigma_{i,j}^2(\epsilon^{2\sigma^2} - 1)} \right) + 1 \right) \\
\mu_{agg} &= \ln \left( \sum_{j=1}^{S} \sum_{i=1}^{N_j} \left( e^{\mu_{i,j} + \frac{\sigma_{i,j}^2}{2}} \right) - \frac{\sigma_{agg}^2}{2} \right),
\end{align*} \quad (8)$$

where $\mu_{i,j}$ and $\sigma_{i,j}^2$ denote the mean and variance of individual summand. Similarly, $\mu_{agg}$ and $\sigma_{agg}^2$ are mean and variance of the resulting $I_{agg}$ distribution.

The above equations are valid for the natural logarithm, and they must be scaled appropriately when working with logarithms to the base of different values ($\log_{10}$ in our case).

**C. Maximum Number of Permissible SUs**

Here, we formulate an optimization problem that allows TEOS to compute the maximum number of SUs that can be safety allowed to operate in each SWS sector. While the objective is to maximize the spatial sharing opportunities
Algorithm 1, which is an approach for identifying spatial IU from aggregate interference caused due to SUs. To evaluate opportunities around an IU while ensuring protection of the performance of TESSO in identifying spatial spectrum sharing to evaluate TESSO. In both studies, we analyze the per-
comparison of the computational complexity of Algorithm 1 
unlike ITM-PTP whose computation complexity grows propor-
TESSO is scalable because its computation time is a constant, 
the precise geolocations of SUs are not available. Moreover,
the above optimization problem can be readily solved using Genetic Algorithms [26].
Recall that Algorithm 1 and TESSO are two different 
approaches for identifying and evaluating the spatial sharing opportunities around an IU. While Algorithm 1 is computa-
tionally expensive (because it is based on ITM-PTP model) and requires the precise geolocations of SUs, TESSO is computationally efficient and works effectively even when the precise geolocations of SUs are not available. Moreover, TESSO is scalable because its computation time is a constant, unlike ITM-PTP whose computation complexity grows proportionally with the number of SUs. We shall provide a detailed comparison of the computational complexity of Algorithm 1 and TESSO in Section V-C.

V. EVALUATION OF TESSO

In this section, we conduct two independent case studies to evaluate TESSO. In both studies, we analyze the performance of TESSO in identifying spatial spectrum sharing opportunities around an IU while ensuring protection of the IU from aggregate interference caused due to SUs. To evaluate the relative effectiveness of TESSO, we compare it against Algorithm 1, which is an approach for identifying spatial
A. Case Study 1: Norfolk Region

As explained above, first, we use Algorithm 1 to compute the total number of SUs, $N$, that can be safely allowed to transmit in the SWS region shown in Figure 1(c). The IU protection criteria is defined by Inequality (1) where $I_{th} = -120$ dBm and $\epsilon = 0.1$. Then, multiple instances of Algorithm 1 are run to obtain the empirical distributions of $I_{agg}$, $N$, and ASC. Figure 3 summarizes the results. As expected, the distribution of $I_{agg}$ shows that the probabilistic guarantee of interference protection to the IU is satisfied. From the plots, we can also observe that $N$ and ASC have skewed Gaussian distributions with mean values of 41.50 and 36.70 bps/Hz respectively.

Next, we evaluate the performance of TESSO in finding the spatial sharing opportunities in the SWS sectors. Let us define a sectorized SWS as shown in Figure 2 which approximately covers the SWS region of Figure 1(c). First, we use Equation (5) to characterize the distribution of $I_{SU}$ in each SWS sector. The values of $\gamma$ and $\sigma$ for each SWS sector are estimated by using samples of the true path loss values (in the absence of measurement data, we assume that ITM-PTP path loss is the true path loss). Then, the distribution of $I_{SU}$ is approximated as a log-normal distribution. Finally, the optimization problem defined in (10) is solved to obtain an optimal value of $N$ using Matlab’s genetic algorithm solver.

Figure 4(a) shows that the log-normal approximation for the distribution of $I_{SU}$ in a SWS sector closely resembles its true distribution (obtained by using ITM-PTP path loss values). The log-normal approximation for $I_{SU}$ distribution is obtained by first finding the weighted least-squares (WLS) values of $\gamma$ and $\sigma$ (parameters of the simplified path loss model) using the true path loss samples, and then using Theorem 1. For computing $\gamma$ and $\sigma$ using WLS, TESSO assigns large weights to smaller path loss values. Doing so ensures that the log-normal approximation matches the tail region of the true distribution of $I_{agg}$.

In Figure 4(b), we compare the true distribution of $I_{agg}$ against the one predicted by TESSO. The true distribution is obtained by using the solution of optimization problem given in Equation (10) and the true path loss values (obtained from the ITM-PTP model). The plots show that TESSO accurately approximates the true distribution. More importantly, TESSO satisfies the IU’s protection requirement.

Figure 4(c) demonstrates the effectiveness of TESSO in identifying SWS opportunities. To make a fair comparison between TESSO and the ITM-PTP model, we add an additional constraint in (10) that ensures the same density of SUs in all SWS sectors. On average, TESSO identifies spatial sharing opportunities ($N$) almost as effectively as the ITM-PTP model (Algorithm 1). Furthermore, comparing the performance in terms of ASC, the plot shows that the ASC achieved by TESSO is comparable to that achieved by the ITM-PTP model.

The difference in the performances between TESSO and the ITM-PTP model is mainly because the ITM-PTP model exploits sharing opportunities throughout the SWS region (see Figure 1(c)), whereas TESSO identifies SWS opportunities only inside the SWS sectors (see Figure 2). Despite this slight disadvantage, TESSO’s lighter computational cost makes it a favorable choice in applications where the geolocations of SUs are not precisely known, and the computation of aggregate interference power needs to be performed in real time for facilitating spectrum access control—such as the case in SAS-driven spectrum sharing.

B. Case Study 2: Fort Green Region

We continue to evaluate TESSO by repeating our analysis in case study 2. Similar to case study 1, first, we generate path loss maps and define the SWS region, as shown in Figures 5(a), 5(b) and 5(c). Then, Algorithm 1 is implemented to compute $N$ and ASC while satisfying IU’s protection requirement. Here, we set $I_{th}$ to a different (compared to case study 1), but arbitrarily chosen, value of $-118$ dBm. Note...
that we chose different $I_{th}$ values in these case studies for representing two different incumbent protection requirements. All other parameters remain the same as in case study 1. The results of Algorithm 1 are summarized in Figure 7. As expected, the plot of $I_{agg}$ shows that the IU’s protection criteria is satisfied. The mean values of $N$ and ASC obtained by Algorithm 1 are 102.61 and 177.05 bps/Hz respectively.

Next, SWS sectors are defined (see Figure 6) which approximately cover the SWS region of Figure 5(c). Then, using samples of true path loss values, the parameters $\gamma$ and $\sigma$ for each sector are estimated, and the distribution of $I_{SU}$ is approximated as a log-normal distribution. Finally, TESSO optimization problem defined by Equation (10) is formulated and solved. The results are summarized in Figure 8.

Plots in Figure 8(a) show that the distribution of $I_{SU}$ can be approximately represented as a log-normal distribution. This approximated distribution is used by TESSO to characterize $I_{agg}$ and to evaluate spatial sharing opportunities. Figure 8(b) shows that TESSO predicts $I_{agg}$ fairly accurately and protects the IU from aggregate interference caused due to SUs. The IU protection requirement of $I_{agg} = -118$ dBm and $\epsilon = 0.1$ is reliably met.

The effectiveness of TESSO in enabling spatial sharing opportunities can be observed in Figure 8(c). The performance of TESSO in estimating $N$ and ASC, is comparable to that of Algorithm 1 (which uses ITM-PTP). As explained in case study 1, the slight difference in the performances of TESSO and Algorithm 1 is mainly because the latter enables spectrum sharing in the entire SWS region, whereas the former enables spectrum sharing only in the SWS sectors. In other words, TESSO slightly under-performs Algorithm 1 because the total area of the SWS sectors is smaller than the total area of the SWS region.

C. Computational Complexity

Note that Algorithm 1, on average, requires $N$ ITM-PTP path loss computations. Therefore, the time complexity of Algorithm 1 is $O(N \times \tau)$, where $O(\tau)$ is the time complexity of each ITM-PTP path loss computation (approx. 100 milliseconds in our implementation). On the other hand, TESSO’s time complexity is constant, and it is the time taken to solve the optimization problem given by (10). In general, time complexity of a genetic algorithm is difficult to express mathematically as it depends on several factors such as population size, crossover type, fitness function, etc. In our simulations, we observed that it takes approximately one second to solve the optimization problem given in (10) using Matlab’s genetic algorithm solver.

TESSO has a clear advantage over Algorithm 1 in terms of computation overhead, especially in cases when $N$ is large. Please refer to Table II for comparison based on our case studies. TESSO takes one second, on average, to solve
the optimization problem given in Equation (10), whereas Algorithm 1 takes \(0.1 \times N\) seconds. The value of \(N\) can be significantly large when IU does not have a very stringent interference protection requirement and SUs’ transmit power is low (e.g., IoT applications, femtocells, etc.). Apart from this computational advantage, TESSO, unlike Algorithm 1, does not require knowledge of the precise locations of SUs in its computations. As long as TESSO knows that the SUs operate inside a given SWS sector, TESSO can evaluate spatial spectrum sharing opportunities reliably.

VI. CONCLUSION

In this paper, we proposed an analytical tool—namely TESSO—that can be used for characterizing the SUs’ aggregate interference and identifying SWS opportunities in dynamic spectrum sharing. TESSO identifies SWS opportunities in a computationally efficient manner without requiring precise geo-locations of secondary users. Our detailed analysis provides the following important insight: An analytical tool, such as TESSO, can be used to exploit SWS opportunities almost as effectively as the terrain-based models, such as the ITM-PTP model. TESSO is computationally efficient, and it provides the same level of interference protection guarantee to the IU compared to that offered by the ITM-PTP model.

REFERENCES


